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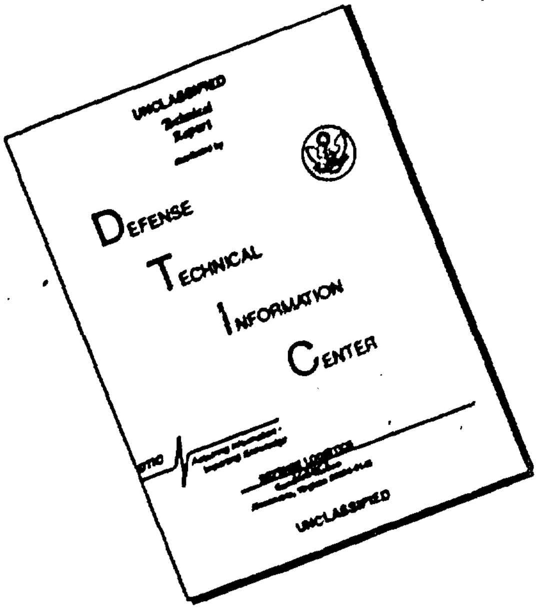

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Lambert's Equations Revisited

22 JULY 1963

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Systems Research and Planning Division

Prepared for COMMANDER SPACE SYSTEMS DIVISION
UNITED STATES AIR FORCE
Inglewood, California

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AEROSPACE CORPORATION
2400 East El Segundo Boulevard
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ABSTRACT

Lambert developed equations relating times of transit between two points in space and the semimajor axis of conics passing through these two points when the two radii and the chord are given. Special types of problems can often best be solved by alternate methods that have been developed, but for a general study of connecting two points in space with a conic section, with no special constraints other than time, Lambert's equations seem to be best suited. This paper represents an expository summary of the mathematical methods and techniques involved.

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I. INTRODUCTION

Statement of Problem

Given two positions in space and their distances from a central body, find an equation that relates time of transit with a geometrical orbital parameter; namely the semimajor axis. The solution to this problem is of interest in orbit determination[1] since knowledge of positions and times are usually involved.

Kepler derived an equation which relates time from perifocal passage with semimajor axis and eccentricity, given the eccentric anomaly of a point. This equation is most often used to find the eccentric anomaly when time, semimajor axis, and eccentricity are known (as is the case in ephemeris prediction). It could be used in a "direct" solution, that is, finding the time, given the other quantities (as is the case in finding the time of change of phase in the patched-conic method of trajectory analysis).

Lambert studied the problem stated and found that by proper substitutions in Kepler's equations he could eliminate the eccentricity and so derive an equation relating time and semimajor axis when two radii and the chord are given. Other methods of orbit determination have also been developed, such as the Lagrange, Gauss, and Gibbs methods or combinations and modifications of them. Which method to use for a given set of known data is a long study in itself and beyond the scope of this paper. Lambert's method is completely general and the mathematical method remains valid for problems involving long arcs. Thus, for problems where the length of the arc is unknown (and could be very long) this method is extremely useful. However, for certain types of problems there is no doubt that other methods can often be more efficient.

II. BASIC GEOMETRICAL RELATIONS

Given the attracting focus, there exist an infinite number of conic sections passing through two given points in space. If in addition, a semimajor axis is given, Table 1 shows the necessary and sufficient conditions to classify the type of resulting conic sections.

Table 1. Classification of Conics.

If:	Then:
$4a > r_d + r_a + c$	2 ellipses
$4a = r_d + r_a + c$	1 ellipse
$4a < r_d + r_a + c$	no ellipses
$4a > c - r_d - r_a$	2 hyperbolas (concave branch)
$4a < r_d + r_a - c$	2 hyperbolas (convex branch)

Figures 1, 2, and 3 illustrate the elliptic cases, Figures 4 and 5 illustrate the hyperbolic cases, and Figure 6 illustrates the parabolic case.

Parabolic and hyperbolic solutions are of interest for a) trajectories to the moon, b) some satellite intercept problems, and c) trajectories of some comets in the solar system.

III. DERIVATION OF LAMBERT'S EQUATION

Lambert's equation may be derived directly from Kepler's equation. The basic formulae and substitutions involved are as follows:[2]

$$r = a(1 - e \cos F) \tag{1}$$

$$r_d + r_a = 2a\left[1 - e \cos \frac{1}{2}(F_a + F_d) \cos \frac{1}{2}(F_a - F_d)\right] \tag{2}$$

or, if

and

$$\left.\begin{aligned} 2G &= F_a + F_d \\ 2g &= F_a - F_d \end{aligned}\right\} \tag{3}$$

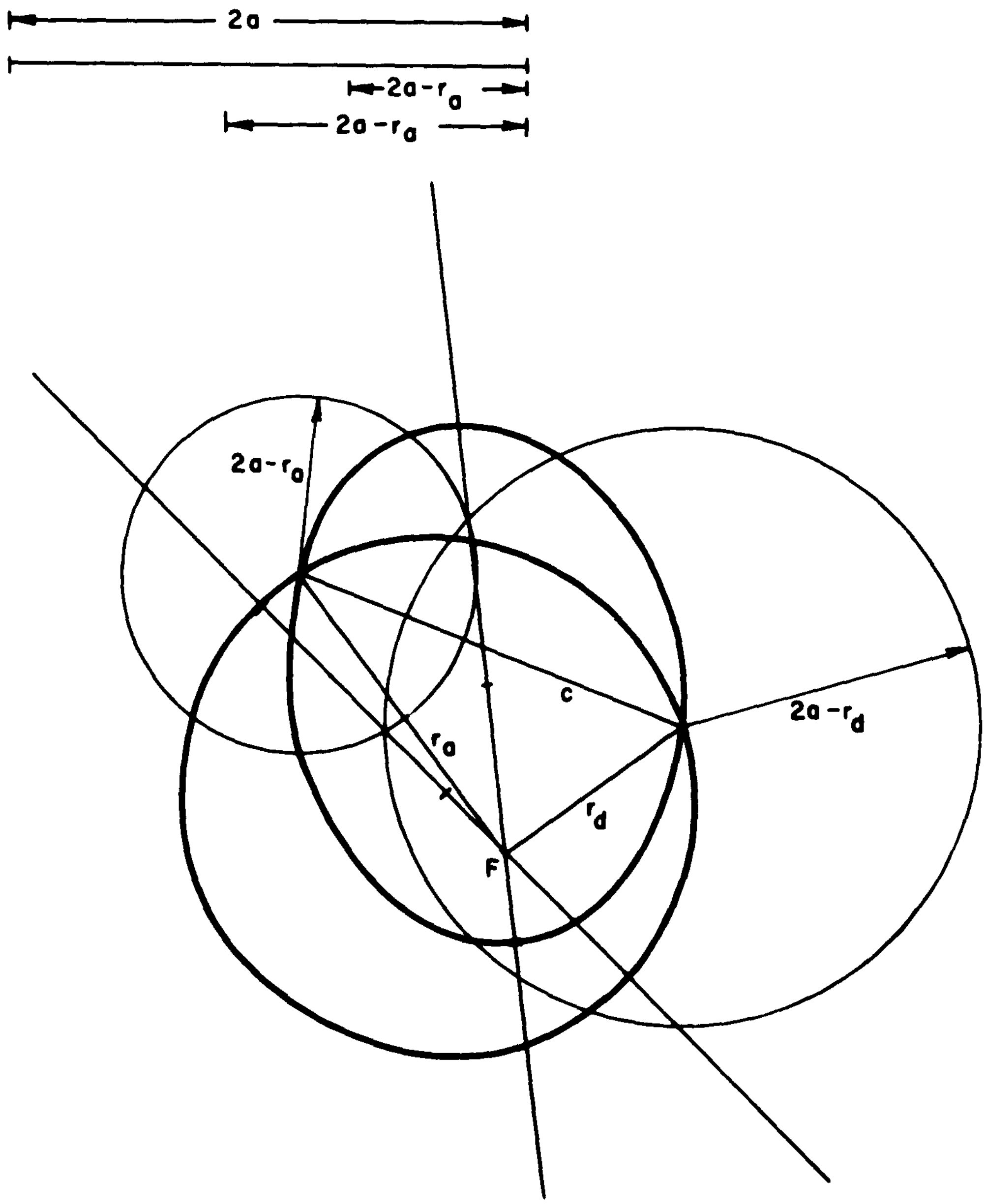


Figure 1. Elliptic Case: $4a > r_d + r_a + c$.

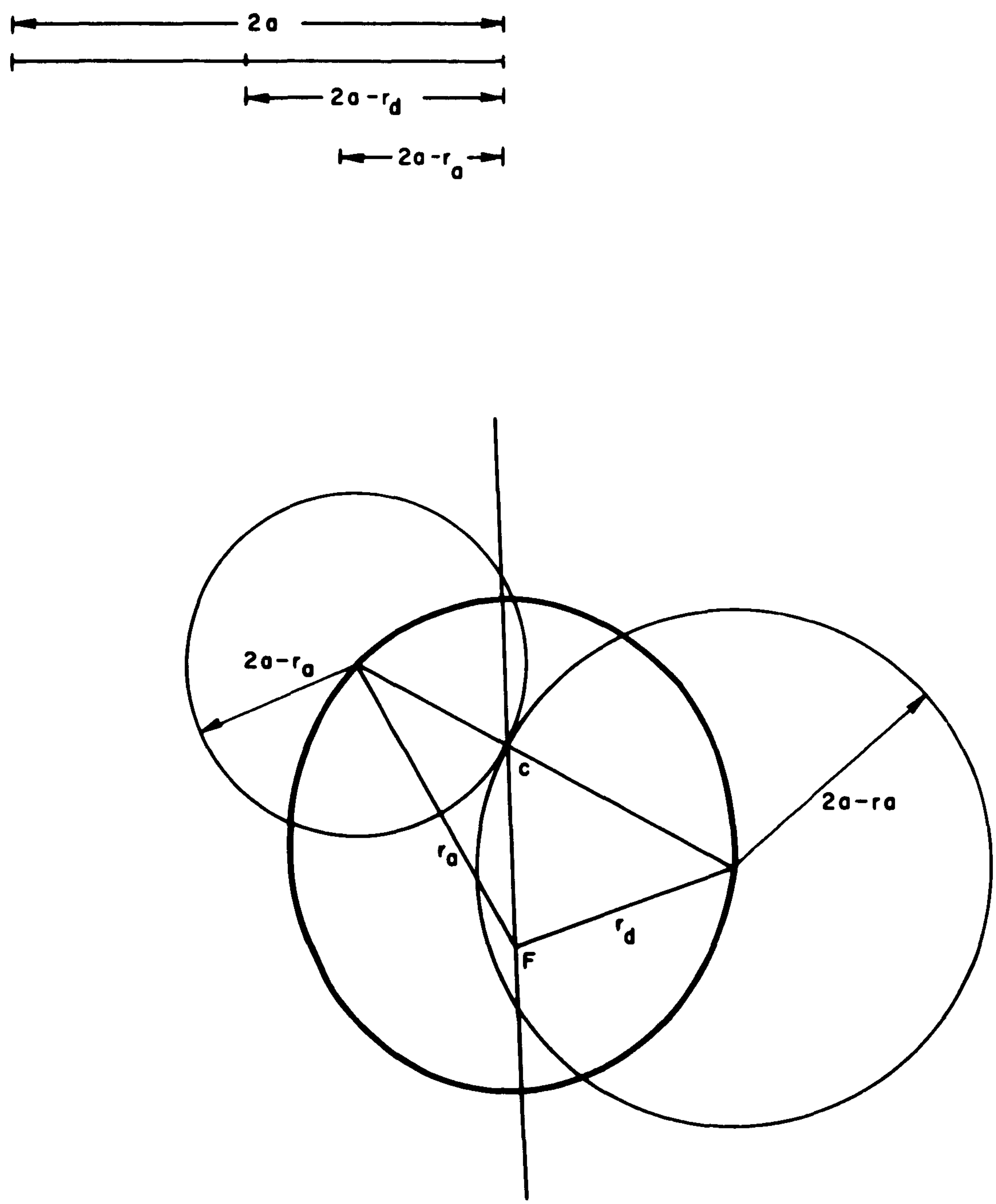
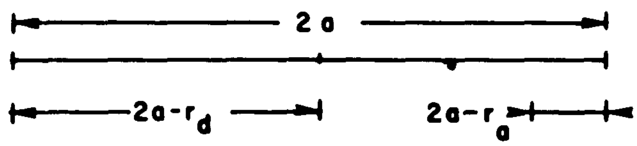


Figure 2. Elliptic Case: $4a = r_d + r_a + c$.



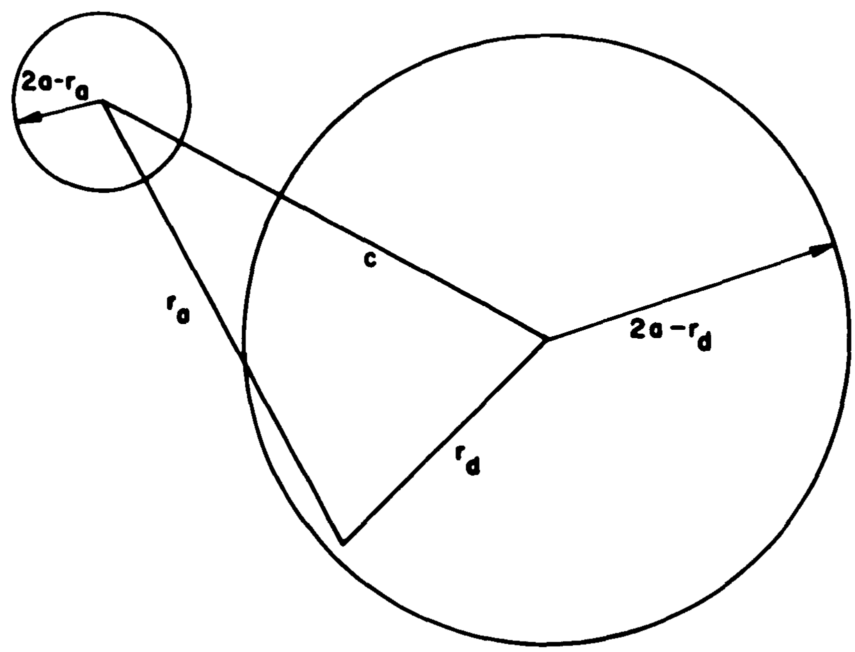


Figure 3. Elliptic Case: $4a < r_d + r_a + c$.

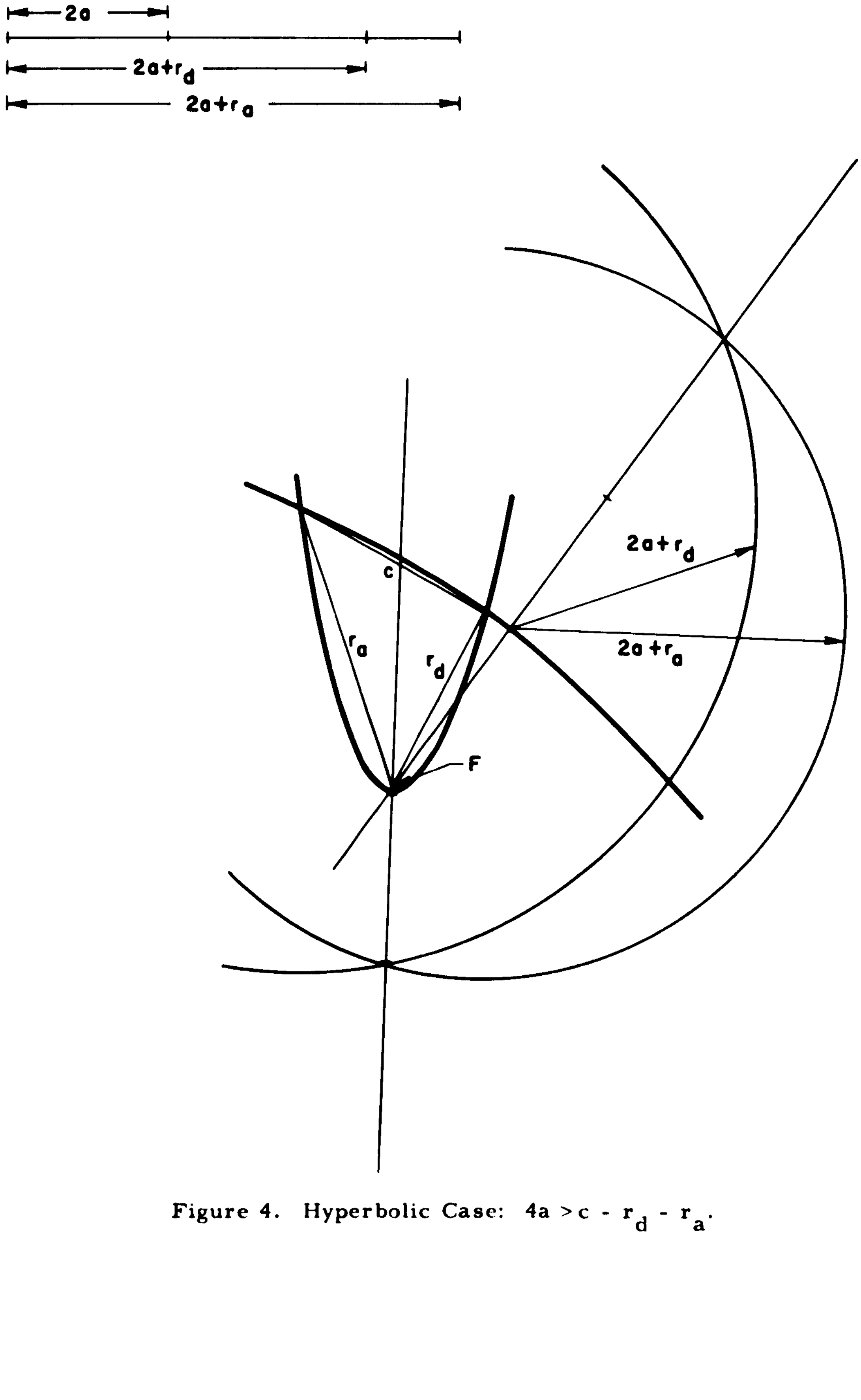


Figure 4. Hyperbolic Case: $4a > c - r_d - r_a$.

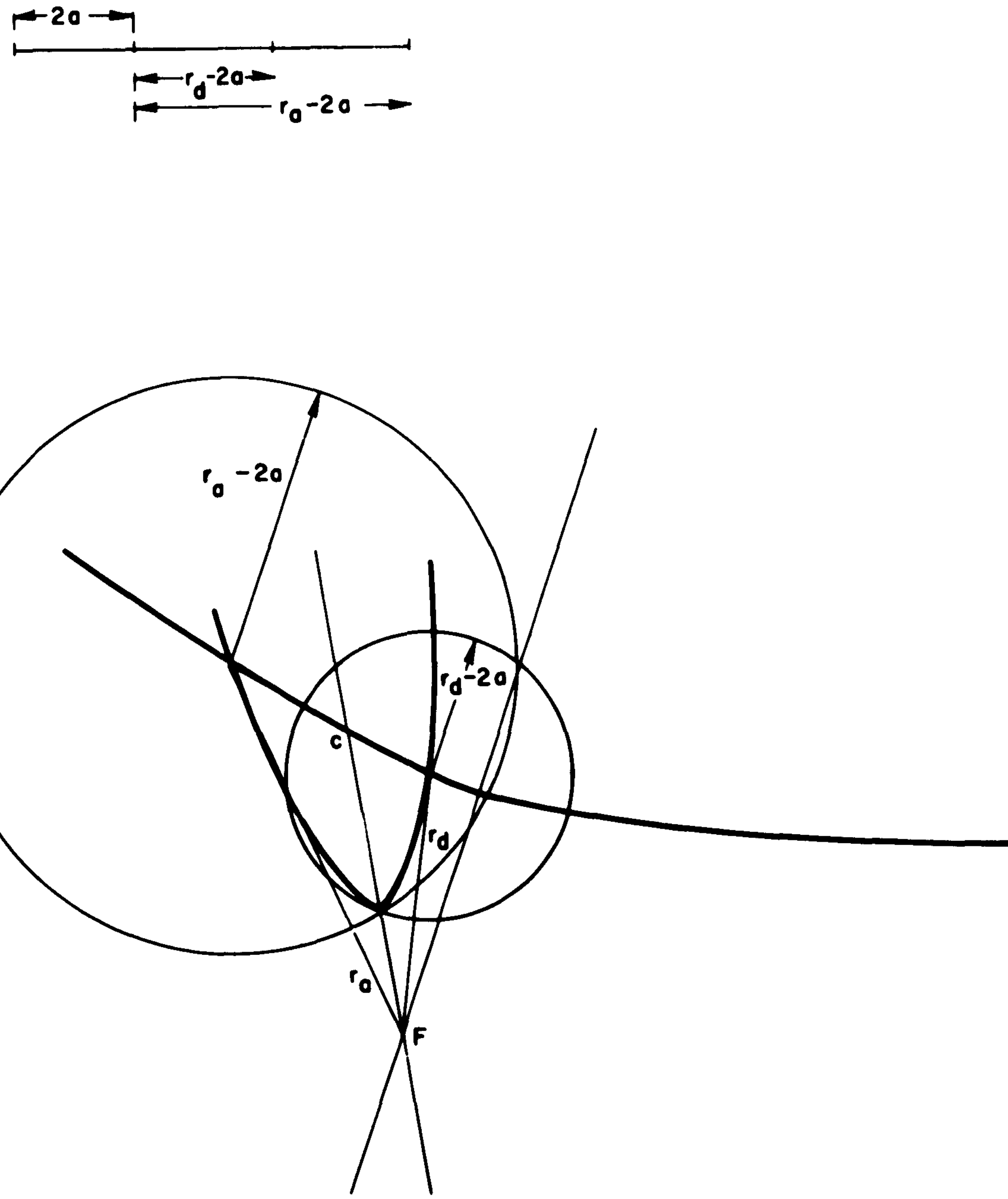


Figure 5. Hyperbolic Case: $4a < r_d + r_a - c$.

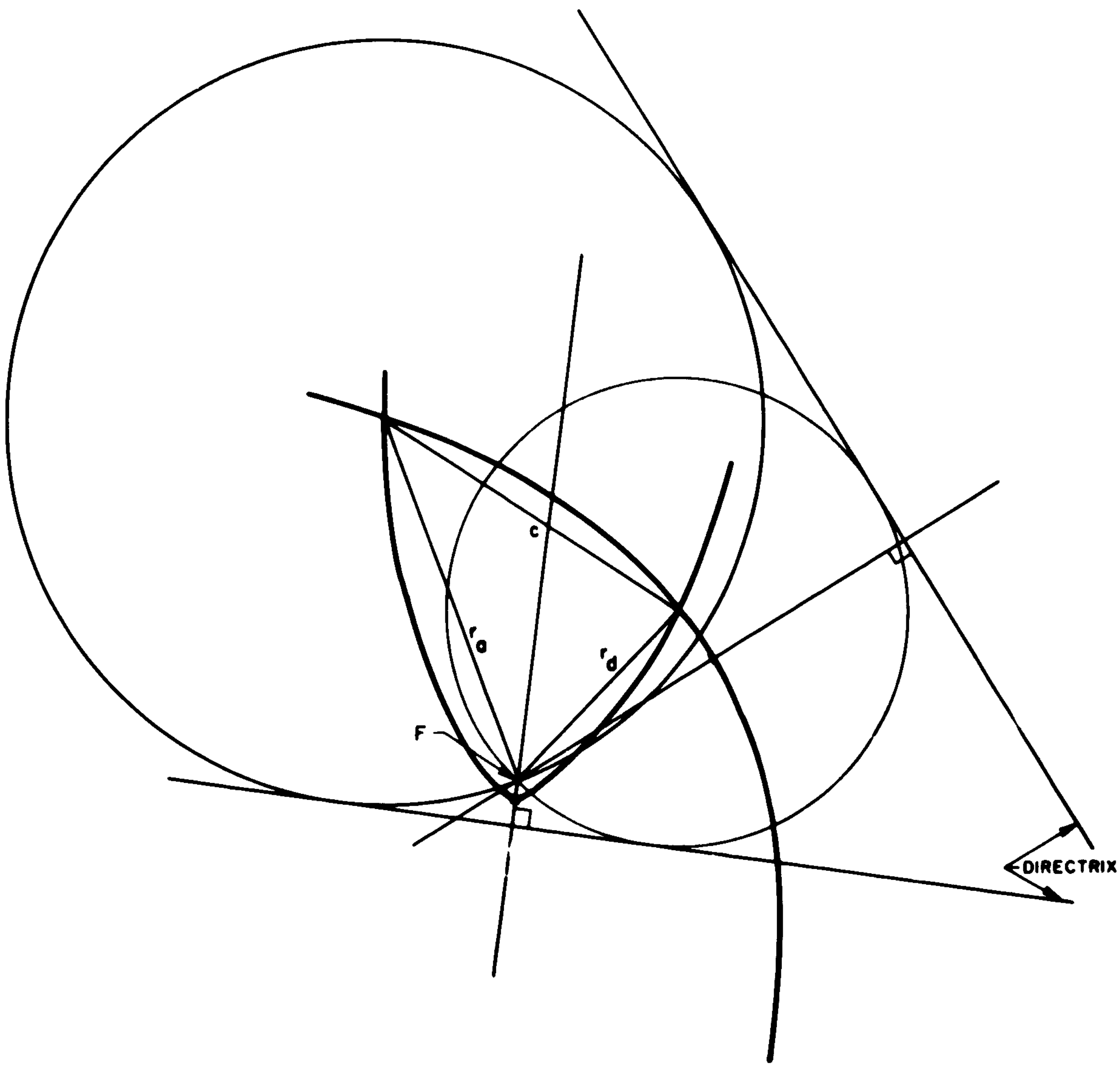


Figure 6. Parabolic Case.

then

$$r_d + r_a = 2a\,(1 - e \cos G \cos g)\ . \tag{4}$$

Similarly,

$$c^2 = 4a^2 \sin^2 G \sin^2 g + 4a^2 (1 - e^2) \cos^2 G \sin^2 g \tag{5}$$

from

$$c^2 = r_d^2 + r_a^2 - 2r_d\, r_a \cos \theta\ . \tag{6}$$

Now let

$$\cos h = e \cos G \tag{7}$$

so that

$$c = 2a \sin g \sin h \tag{8}$$

$$r_d + r_a = 2a(1 - \cos g \cos h) \tag{9}$$

and let

$$\left.\begin{aligned} \epsilon &= h + g \\ \delta &= h - g \end{aligned}\right\} \tag{10}$$

or

$$\left.\begin{aligned} \epsilon - \delta &= F_a - F_d \\ \cos \tfrac{1}{2}(\epsilon + \delta) &= e \cos \tfrac{1}{2}(F_a + F_d) \end{aligned}\right\} \tag{11}$$

then

$$r_d + r_a + c = 4a \sin^2 \tfrac{1}{2}\epsilon \tag{12}$$

$$r_d + r_a - c = 4a \sin^2 \tfrac{1}{2}\delta \tag{13}$$

and substituting in Kepler's equation

$$nt_f = F_a - F_d - e(\sin F_a - \sin F_d) \tag{14}$$

we obtain

$$nt_f = (\epsilon - \sin \epsilon) - (\delta - \sin \delta) \tag{15}$$

which is Lambert's equation.

Since we may assume that $F_a - F_d < 2\pi$, it follows that $0 < (1/2)(\epsilon - \delta) < \pi$ and $0 < (1/2)(\epsilon + \delta) < \pi$, so that $0 < (1/2)\epsilon < \pi$ and $-(1/2)\pi < (1/2)\delta < (1/2)\pi$. Hence Lambert's equation has four possible solutions; that is, combinations of each of the two solutions for ϵ and δ. Let ϵ_1 and δ_1 be their smallest positive values. Then the four solutions are:

Case 1A $$nt_f = (\epsilon_1 - \sin \epsilon_1) - (\delta_1 - \sin \delta_1) \tag{16}$$

Case 2A $$nt_f = (\epsilon_1 - \sin \epsilon_1) + (\delta_1 - \sin \delta_1) \tag{17}$$

Case 1B $$nt_f = 2\pi - (\epsilon_1 - \sin \epsilon_1) - (\delta_1 - \sin \delta_1) \tag{18}$$

Case 2B $$nt_f = 2\pi - (\epsilon_1 - \sin \epsilon_1) + (\delta_1 - \sin \delta_1) \tag{19}$$

which correspond to the four possible times of transfer between the two points in the ellipses of Figure 1.

Call θ the angle measured counterclockwise from r_d to r_a. Then Table 2 indicates whether the motion is direct or retrograde.

Table 2. Direction of Motion, Elliptic.

and/if	$0 < \theta < \pi$	$2\pi > \theta > \pi$
Case 1A	Direct	Retrograde
Case 2A	Retrograde	Direct
Case 1B	Direct	Retrograde
Case 2B	Retrograde	Direct

The equations corresponding to motion along the concave branch of a hyperbola are developed in the same fashion except for the use of hyperbolic in place of trigonometric functions:

$$r = a(e \cosh F - 1) \tag{20}$$

$$\left.\begin{aligned} (\epsilon - \delta) &= (F_a - F_d) \\ \cosh \tfrac{1}{2}(\epsilon + \delta) &= e \cosh \tfrac{1}{2}(F_a + F_d) \end{aligned}\right\} \tag{21}$$

$$r_d + r_a + c = 4a \sinh^2 \tfrac{1}{2}\epsilon \tag{22}$$

$$r_d + r_a - c = 4a \sinh^2 \tfrac{1}{2}\delta \tag{23}$$

$$nt_f = e(\sinh F_a - \sinh F_d) - (F_a - F_d) \tag{24}$$

and finally

$$nt_f = (\sinh \epsilon - \epsilon) - (\sinh \delta - \delta) \tag{25}$$

The above equations show that ϵ is always positive. Furthermore if the angle θ, as defined above, is less than π then $\delta > 0$ and if $\theta > \pi$, then $\delta < 0$. Therefore two cases exist for hyperbolic motion:

$$\text{Case 1C} \quad nt_f = (\sinh \epsilon - \epsilon) - (\sinh \delta - \delta) \tag{26}$$

$$\text{Case 2C} \quad nt_f = (\sinh \epsilon - \epsilon) + (\sinh \delta - \delta) \tag{27}$$

and Table 3 tabulates the direction of motion:

Table 3. Direction of Motion: Hyperbolic.

and/if	$0 < \theta < \pi$	$2\pi > \theta > \pi$
Case 1C	Direct	Retrograde
Case 2C	Retrograde	Direct

Since motion along the convex branch of a hyperbola is of no practical interest it may properly be omitted.

The equation for motion along a parabola was found by Euler, and for completeness it will be included here. Euler's equation follows from Baker's equation[3] (the counterpart of Kepler's equation for a parabola):

$$\frac{\sqrt{\mu}\,(t - \tau)}{\sqrt{2}\, q^{3/2}} = \tan \frac{v}{2} + \frac{1}{3} \tan^3 \frac{v}{2} \tag{28}$$

Also, for a parabola,

$$r = q \sec^2 \frac{v}{2} = q\left(1 + \tan^2 \frac{v}{2}\right) \tag{29}$$

so that

$$r_d + r_a = q\left(2 + \tan^2 \frac{v_d}{2} + \tan^2 \frac{v_a}{2}\right) \tag{30}$$

and the equation for the chord becomes

$$c^2 = (r_d + r_a)^2 - 4\, r_d\, r_a \cos^2\left(\frac{v_a - v_d}{2}\right) \tag{31}$$

or

$$2\sqrt{r_d\, r_a}\, \cos \frac{(v_d - v_a)}{2} = \pm\sqrt{(r_d + r_a + c)(r_d + r_a - c)} \tag{32}$$

$$1 + \tan \frac{v_d}{2} \tan \frac{v_a}{2} = \pm \frac{\sqrt{(r_d + r_a + c)(r_d + r_a - c)}}{2q} \tag{33}$$

so that

$$\frac{(r_d + r_a + c) + (r_d + r_a - c) \mp 2\sqrt{(r_d + r_a + c)(r_d + r_a - c)}}{2q}$$

$$= \left(\tan \frac{v_a}{2} - \tan \frac{v_d}{2}\right)^2 \tag{34}$$

or

$$\frac{\sqrt{(r_d + r_a + c)} \mp \sqrt{(r_d + r_a - c)}}{\sqrt{2q}} = \tan \frac{v_a}{2} - \tan \frac{v_d}{2} \tag{35}$$

Using equation 28 to find the time of flight between two points in the orbit gives

$$\frac{\sqrt{\mu}\, t_f}{\sqrt{2}\, q^{3/2}} = \tan \frac{v_a}{2} - \tan \frac{v_d}{2} + \frac{1}{3}\left(\tan^3 \frac{v_a}{2} - \tan^3 \frac{v_d}{2}\right) \tag{36}$$

which can also be written as,

$$\frac{\sqrt{\mu}\, t_f}{\sqrt{2}\, q^{3/2}} = \left(\tan \frac{v_a}{2} - \tan \frac{v_d}{2}\right)\left[3\left(1 + \tan \frac{v_d}{2} \tan \frac{v_a}{2}\right) + \left(\tan \frac{v_a}{2} - \tan \frac{v_d}{2}\right)^2\right] \tag{37}$$

Substitution from equations 33 and 35 yields

$$6\sqrt{\mu}\, t_f = (r_d + r_a + c)^{3/2} \mp (r_d + r_a - c)^{3/2} \tag{38}$$

where for direct orbits the upper sign, Case 1D, is used if $\theta < \pi$ and the bottom one, Case 2D, if $\theta > \pi$. The signs are reversed for retrograde orbits.

Table 4. Direction of Motion: Parabolic.

and/if	$\theta < \pi$	$\theta > \pi$
Case 1D	Direct	Retrograde
Case 2D	Retrograde	Direct

IV. APPLICATIONS

As previously mentioned, primary applications of Lambert's equations are in preliminary orbit determination and parametric studies of transfer orbits. This application involves the solution of transcendental equations and the problem is further complicated by the many equations which are involved (equations 16 through 19, 26, 27, and 38). The most practical method for choice of equations to be solved is presented by Breakwell[4] and involves a test on time.

Breakwell's parameters are: a unitless time

$$T = \frac{2\pi t_f}{P_s} \tag{39}$$

where P_s = period of elliptic orbit with semimajor axis $s/2$; a unitless energy

$$E = \frac{\text{energy of transfer orbit}}{E_s} \tag{40}$$

where E_s = energy of elliptic orbit with semimajor axis $s/2$; and a unitless linear scale

$$K = 1 - \frac{c}{s} \tag{41}$$

where

$$s = \frac{1}{2}(r_d + r_a + c) \tag{42}$$

A plot of E versus T for a $K \approx 0.8$ is schematized in Figure 7. When $E = 0$ and $E = -1$, expressions for T in terms of K are easily obtained and become landmarks:

$$T_{1A} = \frac{4}{3}(1 - K^{3/2}) \tag{43}$$

$$T_{2A} = \frac{4}{3}(1 + K^{3/2}) \tag{44}$$

$$T_{1B} = \pi - 2\sin^{-1}\sqrt{K} + 2\sqrt{K}\sqrt{1 - K} \tag{45}$$

$$T_{2B} = \pi + 2\sin^{-1}\sqrt{K} - 2\sqrt{K}\sqrt{1 - K} \tag{46}$$

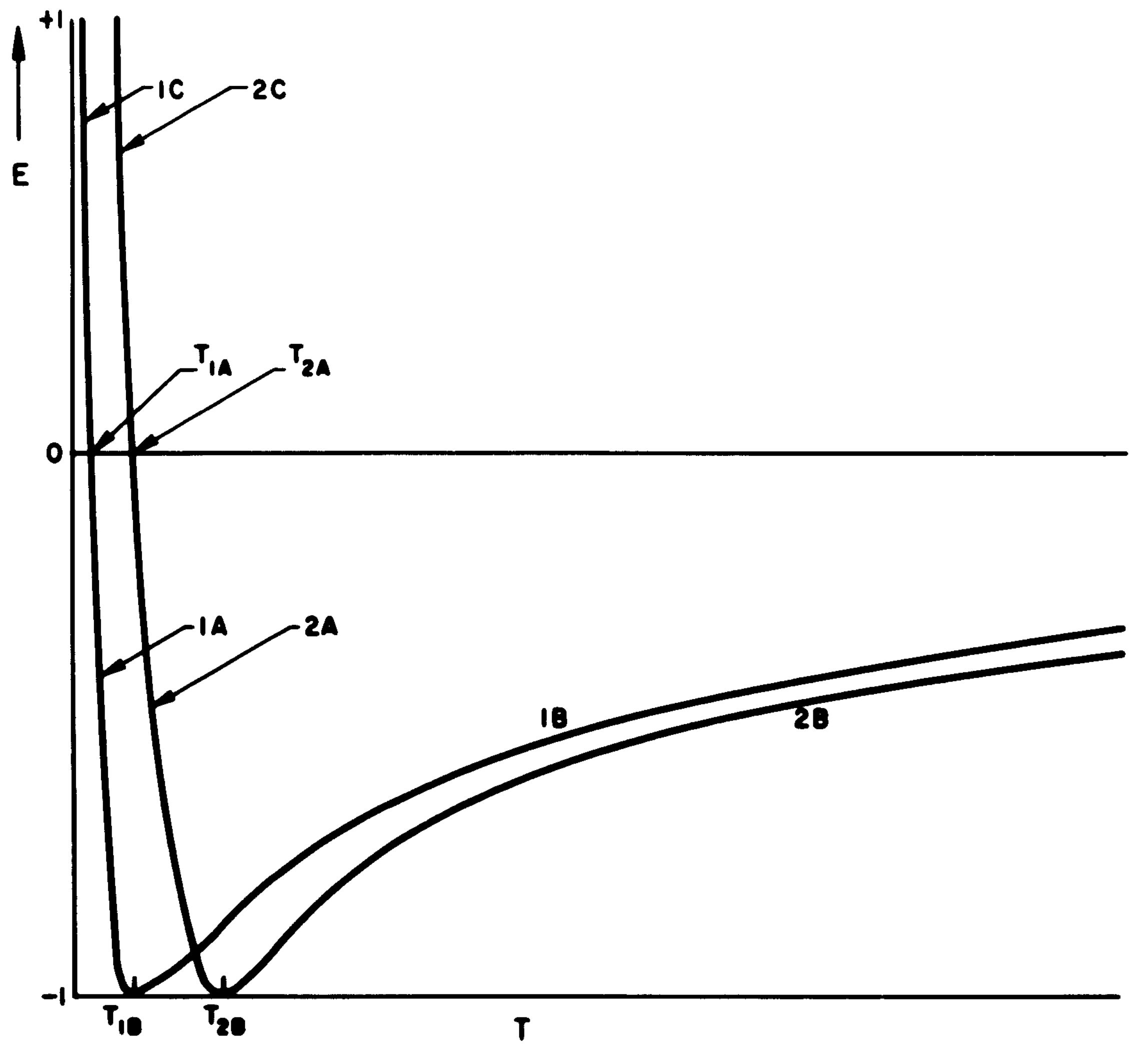


Figure 7. Energy Versus Time.

A study of Figure 7 leads to Table 5 which together with Tables 1 through 4 allows the choice of the proper equation to be used and determines, without further solving the problem, the direction of motion for each case.

Table 5. Classification of Cases According to Time.

If t_f is		Then Use Case
	$< T_{1A}$	1C 2C
	$= T_{1A}$	1D (parabola - no "solution" necessary)
	$> T_{2B}$	1B 2B
	$= T_{2A}$	2D (parabola - no "solution" necessary)
T_{2A} T_{1B}	$> T_{1A}, < T_{1B}$	1A 2C
	$> T_{1B}, < T_{2A}$	1B 2C
	$> T_{2A}, < T_{2B}$	2A 1B
$T_{2A} < T_{1B}$	$> T_{1A}, < T_{2A}$	1A 2C
	$> T_{2A}, < T_{1B}$	1A 2A
	$> T_{1B}, < T_{2B}$	2A 1B

The case of nearly parabolic motion, either from the elliptical or hyperbolic side presents some difficulties. Such cases have been solved by Lancaster[5] where the elliptic (or hyperbolic) equations are replaced by series expansions which converge rapidly for nearly parabolic motion:

$$\text{Case 1NP} \qquad T = - \sum_{n=0}^{\infty} \frac{(-1)\cdot 1\cdot 3\cdot 5\cdots(2n-1)}{2^{n-2}\ n!} \frac{(-E)^n}{2n+3} \left(1 - K^{n+3/2}\right) \tag{47}$$

$$\text{Case 2NP} \qquad T = - \sum_{n=0}^{\infty} \frac{(-1)\cdot 1\cdot 3\cdot 5\cdots(2n-1)}{2^{n-2}\ n!} \frac{(-E)^n}{2n+3} \left(1 + K^{n+3/2}\right) \tag{48}$$

Table 4 will still give the direction of motion, where Case 1NP would replace Case 1D and Case 2NP would replace Case 2D.

Equations (47) and (48) could be inverted but they can be used with iteration as they stand to solve for E when T is given. The previous tables are summarized in Tables 6 and 7.

Table 6. Summary of Cases.

If		Then Use Case	Equation
$t_f \le T_{1A} - \Delta T_1$		1C	(26)
		2C	(27)
$T_{1A} - \Delta T_1 < t_f \le T_{1A} + \Delta T_2$		1NP	(47)
$T_{2A} - \Delta T_3 \le t_f \le T_{2A} + \Delta T_4$		2NP	(48)
$T_{2A} > T_{1B}$	$T_{1A} + \Delta T_2 < t_f \le T_{1B}$	1A	(16)
		2C	(26)
	$T_{1B} < t_f < T_{2A} - \Delta T_3$	1B	(18)
		2C	(27)
	$T_{2A} + \Delta T_4 < t_f \le T_{2B}$	2A	(17)
		1B	(18)
$T_{2A} < T_{1B}$	$T_{1A} + \Delta T_2 < t_f < T_{2A} - \Delta T_3$	1A	(16)
		2C	(27)
	$T_{2A} + \Delta T_4 < t_f \le T_{1B}$	1A	(16)
		2A	(17)
	$T_{1B} < t_f \le T_{2B}$	2A	(17)
		1B	(18)
$t_f > T_{2B}$		1B	(18)
		2B	(19)

Table 7. Summary of Direction of Motion.

and/if	$\varphi < 180$	$\varphi > 180$
Case 1C	Direct	Retrograde
2C	Retrograde	Direct
1NP	Direct	Retrograde
2NP	Retrograde	Direct
1A	Direct	Retrograde
2A	Retrograde	Direct
1AB	Direct	Retrograde
2AB	Retrograde	Direct
1B	Direct	Retrograde
2B	Retrograde	Direct

Expressions for ΔT_1, ΔT_2, ΔT_3, and ΔT_4 are not yet available. In practice an iterative scheme can be used which would "jump" from Cases NP to C or NP to A and proceed to the answer.

Another case which may not be of practical use is when E approaches zero from the 1B or 2B expressions; that is, T approaches infinity. These are taken care of by the following series:

Case 1BP
$$T = \frac{2\pi}{(\sqrt{-E})^3} + \sum \frac{(-1)\cdot 1\cdot 3\cdot 5\cdots(2n-1)}{2^{n-2}\, n!} \frac{(-E)^n}{2n+3} (1 + K^{n+3/2}) \quad (49)$$

Case 2BP
$$T = \frac{2\pi}{(\sqrt{-E})^3} + \sum \frac{(-1)\cdot 1\cdot 3\cdot 5\cdots(2n-1)}{2^{n-2}\, n!} \frac{(-E)^n}{2n+3} (1 - K^{n+3/2}) \quad (50)$$

A relatively simple computer program can be developed which would solve all the above cases in a unified iterative procedure.

The above study is limited to one revolution of the transfer orbit. For more than one revolution all that is required is to add $2N\pi$ to the right hand side of the equations to be solved (where N = number of complete revolutions of the transfer orbit). This may bring in some additional numerical difficulties in the solutions which in turn may produce more special cases. Such cases have yet to be studied.

V. ORBITAL ELEMENTS

Once "a" has been obtained from the solution of Lambert's equations, the rest of the orbital elements may be easily found.[1] For elliptic orbits

$$ea \sin F_d = \frac{(r_a - a) - (r_d - a)\cos(\epsilon - \delta)}{\sin(\epsilon - \delta)} \tag{51}$$

$$ea \cos F_d = a - r_d \tag{52}$$

which gives the eccentricity. Then

$$M_d = F_d - e \sin F_d \tag{53}$$

$$\tau = t_d - \frac{M_d}{n} \tag{54}$$

where

$$n = \frac{\sqrt{\mu}}{|a|^{3/2}} \tag{55}$$

thus obtaining the time of perifocal passage. To find the angular elements, the cartesian coordinates or the direction cosines of the two positions must be known.

$$\cos i = \frac{|L_z|}{L_x^2 + L_y^2 + L_z^2} \tag{56}$$

$$\sin i \sin \Omega = \frac{L_x}{L_x^2 + L_y^2 + L_z^2} \tag{57}$$

$$-\sin i \cos \Omega = \frac{L_y}{L_x^2 + L_y^2 + L_z^2} \tag{58}$$

where

$$\left.\begin{aligned} L_x &= y_1 z_2 - z_1 y_2 \\ L_y &= z_1 x_2 - x_1 z_2 \\ L_z &= x_1 y_2 - y_1 x_2 \end{aligned}\right\} \tag{59}$$

and $0 \leq i \leq \pi/2$ for direct orbits and $\pi/2 < i < \pi$ for retrograde orbits. Finally, the argument of perifocus, ω, is:

$$\omega = u_1 - v_1 \tag{60}$$

$$\frac{r_d}{a} \cos v_d = \cos F_d - e \tag{61}$$

$$\frac{r_d}{a} \sin v_d = \sqrt{|1 - e^2|} \sin F_d \tag{62}$$

where v_d and u_d may be obtained from

$$\left.\begin{aligned} r_d \cos u_d &= x_d \cos \Omega + y_d \sin \Omega \\ r_d \sin u_d &= -x_d \cos i \sin \Omega + y_d \cos i \cos \Omega + z_d \sin i \end{aligned}\right\} \tag{63}$$

For hyperbolic orbits, the following substitutions will suffice

1. Use hyperbolic functions for the eccentric anomaly (remember that sin → - sinh)

2. $M_d = -F_d + e \sinh F_d$

For parabolic orbits

$$a = \infty$$

$$e = 1$$

and for time of perifocal passage use Baker's equation:

$$M_d = \tan \frac{v_d}{2} + \frac{1}{3} \tan^3 \frac{v_d}{2} \tag{64}$$

where

$$\left.\begin{aligned} \tan \frac{v_d}{2} &= \frac{\cos \frac{\Delta v}{2} \pm \sqrt{\frac{r_d}{r_a}}}{\sin \frac{\Delta v}{2}} \\ \tan \frac{v_a}{2} &= \frac{-\cos \frac{\Delta v}{2} \pm \sqrt{\frac{r_a}{r_d}}}{\sin \frac{\Delta v}{2}} \end{aligned} \quad (0 \le \frac{v}{2} \le \pi) \right\} \tag{65}$$

$$(\Delta v = +\theta \text{ for direct orbits}$$

$$\Delta v = -\theta \text{ for retrograde orbits})$$

and choose v_d such that $v_a - v_d = \Delta v$. Thus, obtain q from

$$r_d = q\left(1 + \tan^2 \frac{v_d}{2}\right) \tag{66}$$

and finally, τ, from

$$\frac{\sqrt{u}\,(t_d - \tau)}{\sqrt{2}\, q^{3/2}} = M_d \tag{67}$$

The angular elements are obtained as for ellipses and hyperbolas.

NOMENCLATURE

a	semimajor axis
c	magnitude of chord between two radii
e	eccentricity
E	energy ratio
F	eccentric anomaly
i	inclination of orbit plane to reference plane
K	see equation 41
M	mean anomaly
n	mean motion
P	period
q	radius to perifocus
r	magnitude of radius vector
S	semiperimeter
t	time
T	time ratio
v	true anomaly
δ	variable in Lambert's Equation (see equations 12 and 22)
ϵ	variable in Lambert's Equation (see equations 13 and 23)
θ	central angle between the two radii always measured counterclockwise from r_d to r_a regardless of direction of motion
μ	gravitation and mass factor
τ	time of perifocal passage
Ω	longitude (or right ascension) of the node
ω	argument of perifocus

Subindex

d	departure
a	arrival
f	flight

CONTEMPORARY REFERENCES

1. Sconzo, P., suggests a method of inverse interpolation for the solution of Lambert's Equation in the A. J., Vol 67, No. 1, February 1962, pp. 19-21. He then proceeds to write the equations necessary to obtain the rest of the orbital elements.

2. The simplest and most straightforward presentation of Lambert's Equations is in "An Introductory Treatise on Dynamical Astronomy," H. C. Plummer, pp. 47-57. He derives the equations for elliptic, parabolic, and hyperbolic motions. The geometry of the problem is presented, though not illustrated. (Dover Publications, Inc. N. Y., 1960 Edition)

3. Moulton, Forest Ray, "Celestial Mechanics," pp. 157-158. (The MacMillan Co., 1959 Edition)

4. Breakwell, J. V., R. W. Gillespie, and S. Ross, treated the problem for the inverse solution in the ARS Journal of February 1961, pp. 201-208. Here they solve the equations for the semimajor axis when the time of transit is known. A clever unitless set of variables is introduced which aids in the understanding of various possible solutions and in the choice of the proper equation to be solved.

5. Lancaster, E. R., unpublished.

6. Gedeon, G., in Northrop Report NSL 62-13, February 1962, develops Lambert's Equations and then by a transformation writes them in a power series. These have the advantage of being continuous from elliptical though parabolic to hyperbolic motions. The series are divergent in some areas and slowly convergent in others. The inverse problem is presented but not solved. Work in this area is apparently being done but is very difficult. Gedeon's criterion for choosing an equation for a set of given circumstances only takes care of direct orbits, which are the interesting ones in most studies (interplanetary).

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Aerospace Corporation, El Segundo, California. LAMBERT'S EQUATIONS REVISITED by S. R. Marcus. 22 July 1963. 23 pages including illustrations. (Report TDR-169(3550-40)TN-3, SSD-TDR-63-177) Contract No. AF04(695)-169.

Unclassified Report

Lambert developed equations relating times of transit between two points in space and the semi-major axis of conics passing through these two points when the two radii and the chord are given. Special types of problems can often best be solved by alternate methods that have been developed, but for a general study of connecting two points in space with a conic section, with no special constraints other than time, Lambert's equations seem to be best suited. This paper represents an expository summary of the mathematical methods and techniques involved.

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